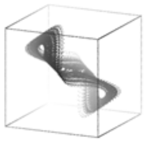

Types of Dynamical Systems

Kevin Dooley

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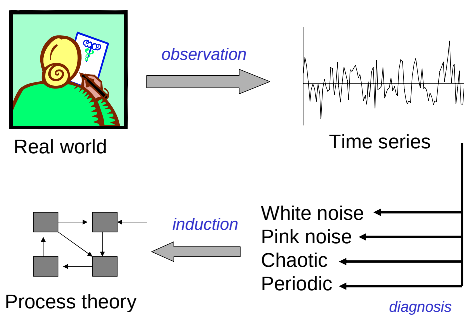
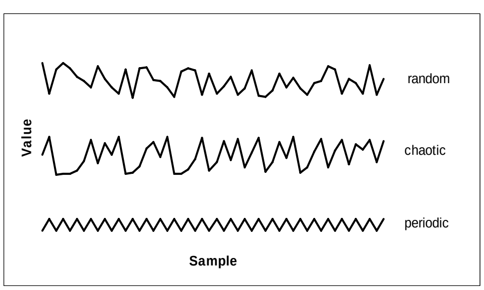
http://www.public.asu.edu/~kdooley/

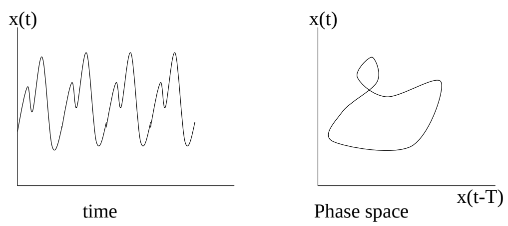
Story telling with historical data



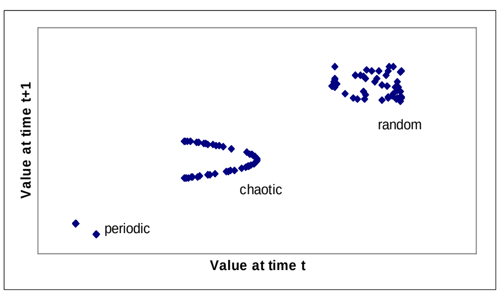
Dynamical systems: Patterns over time



Phase diagrams

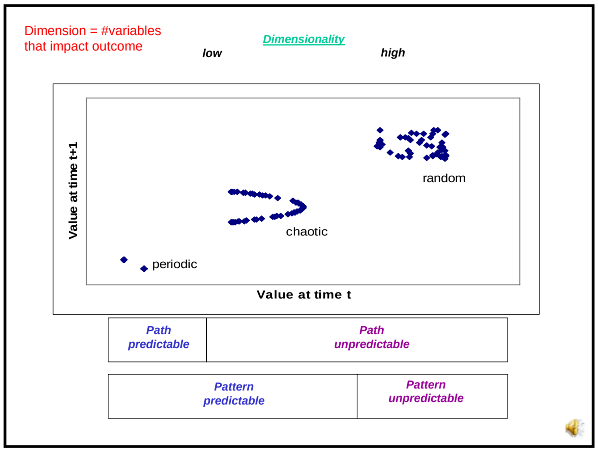


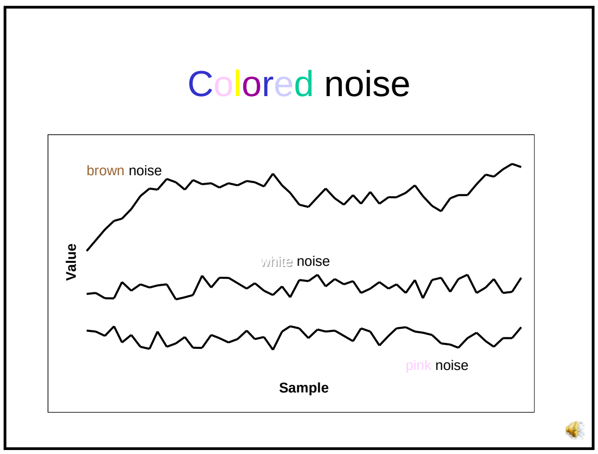
Another view...

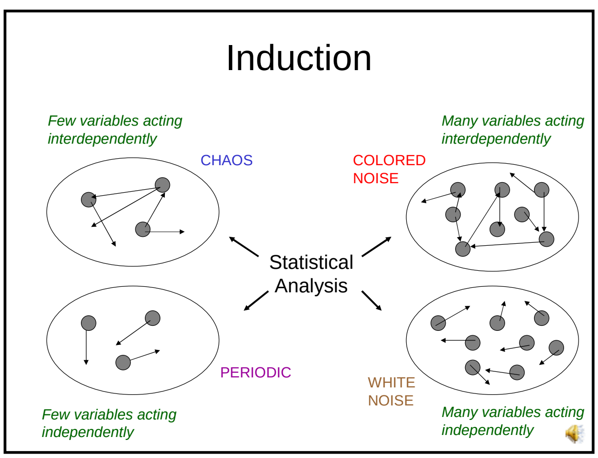


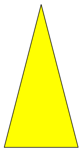
Path v. pattern

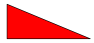
- Path
  - Exact sequence of time series points into the future
- Pattern
  - General boundaries and tendencies of time series behavior


Dimension = #variables
that impact outcome
Dimensionality
low
high
Value at time t+1
random
chaotic
periodic
Value at time t
Path
predictable
Path
unpredictable
Pattern
predictable
Pattern
unpredictable


Colored noise
brown noise
Value
white noise
pink noise
Sample

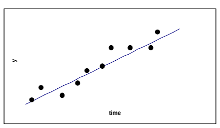

Induction
Few variables acting
interdependently
CHAOS
COLORED
NOISE
Many variables acting
interdependently
Statistical
Analysis
PERIODIC
WHITE
NOISE
Few variables acting
independently
Many variables acting
independently



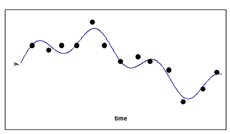


Low dimensional linear systems

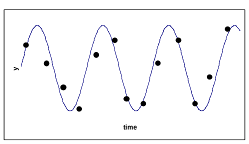
Linear parametric systems



Straight line

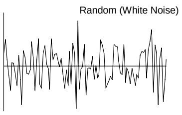


Two sine waves



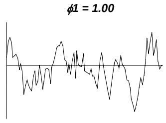
Sine wave

Random (White Noise)



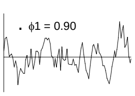
AR(1)

$\phi 1$ = 1.00

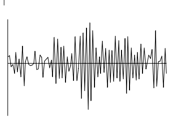


Autoregressive model, order 1:
$x_t = \phi_1 * x_{t-1} + e_t$, e_t is NIID

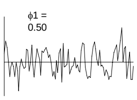
- $\phi 1$ = 0.90



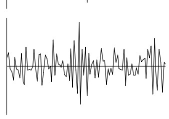
. *$\phi 1$ = -0.90*



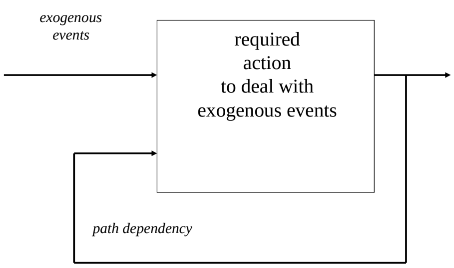
$\phi 1$ =
0.50



$\phi 1$ = -0.50



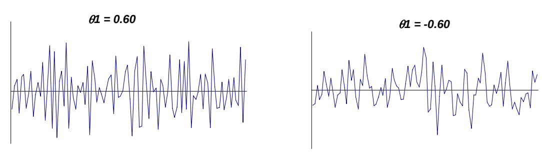
AR(1) model



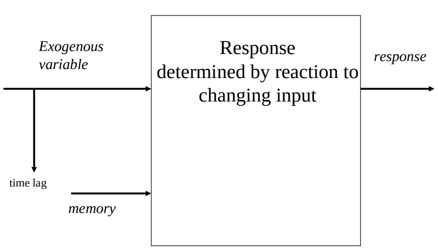
MA (1)

Moving average model, order 1:

$x_t = \theta_1 * e_{t-1} + e_t,$ e_t is NIID

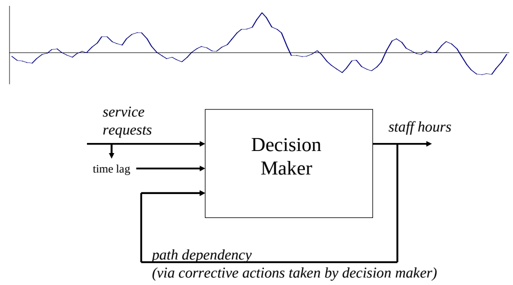


MA(1) model

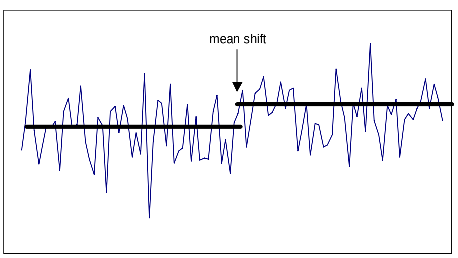


ARMA(2,1)

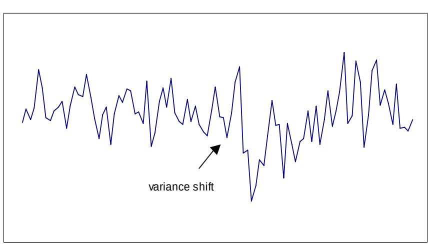
$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + a_t - \theta_1 a_{t-1}$$



Phases and epochs

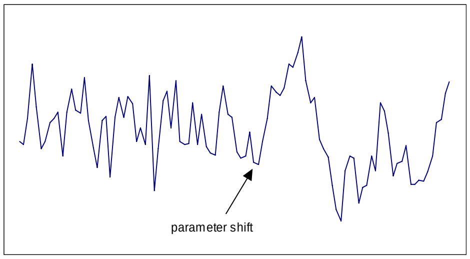


Epochs (2)



AR(1) System with a Shift in Variance

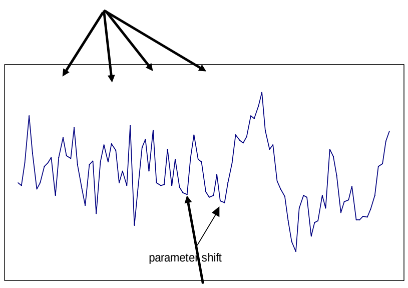
Epochs (3)



AR(1) System with a Shift in Dynamical Parameters

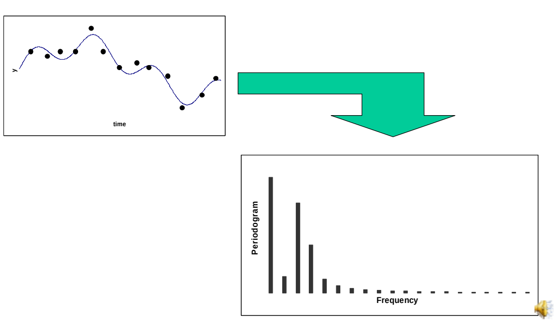
Epochs (4)

unknown: change point detection

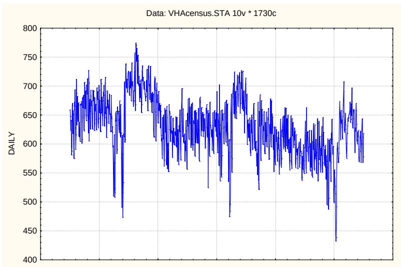


hypothesize a priori: intervention analysis

Periodogram

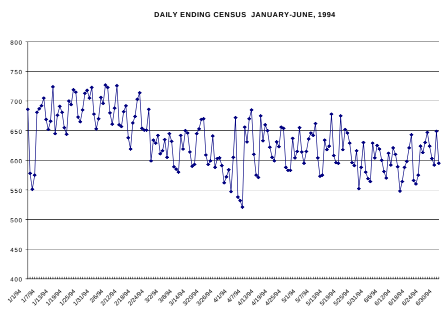


Daily Census (5 Years)

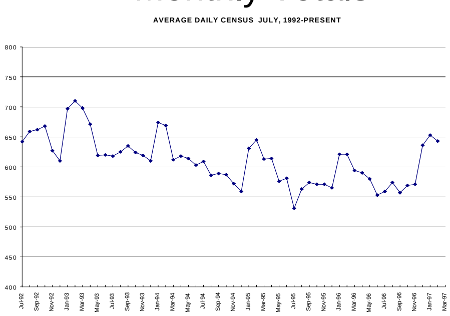


VHA Participating Hospital

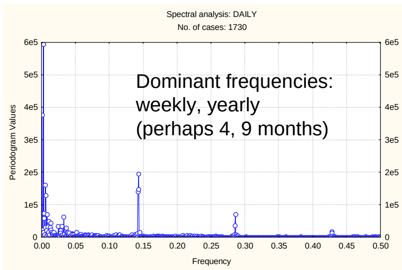
One Year's of Data...

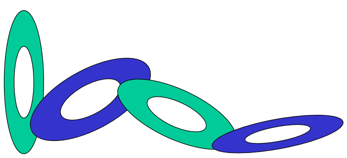


Monthly Totals



Spectral Analysis

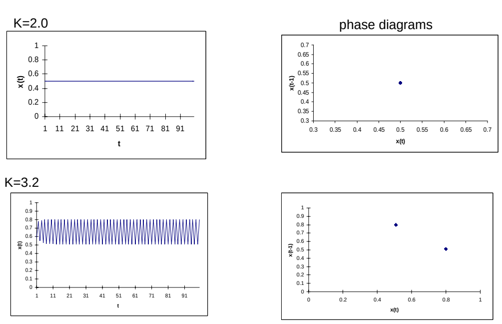




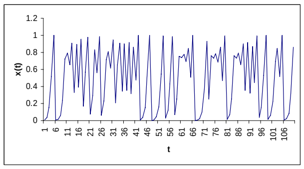
Low dimensional nonlinear systems (chaos)

x(t+1)=k*x(t)*(1-x(t))

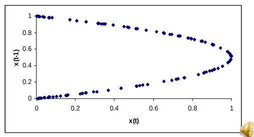
Logistics curve



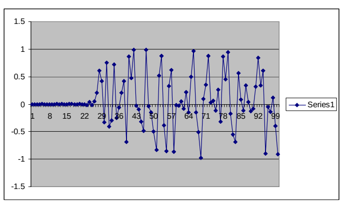
Logistics Curve (2)



K=4



Logistics Equation--Sensitivity



Difference between initial condition of 0.01 and 0.010000001

Sensitivity to Initial Conditions

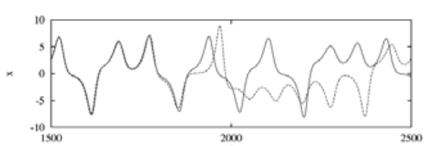


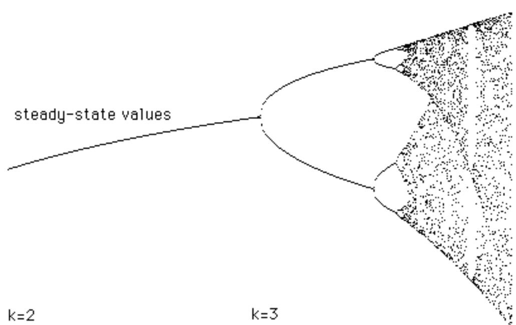
Figure 11.7 Two time evolutions of x with an infinitesimal initial difference

The flapping of a single butterfly's wing today produces a tiny change in the state of the atmosphere. Over a period of time, what the atmosphere actually does diverges from what it would have done. So, in a month's time, a tornado that would have devastated the Indonesian coast doesn't happen. Or maybe one that wasn't going to happen, does. (Ian Stewart, *Does God Play Dice? The Mathematics of Chaos*, pg. 141)

The Butterfly Effect

"For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse, the rider was lost;
For want of a rider, a message was lost;
For want of a message the battle was lost;
For want of a battle, the kingdom was lost!"

Bifurcation Plot



Bifurcation Diagram

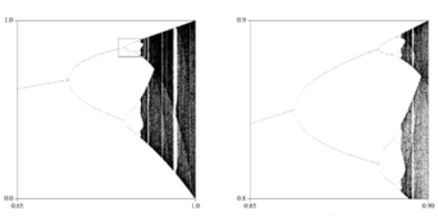
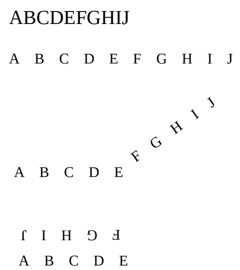


Figure 10.7 Bifurcation diagrams for the logistic map: (a) This image has values of r such that fixed points, limit cycles, and chaos are all visible. (b) This image shows the detail of the boxed section of (a).

Chaos as Stretch & Fold



Stretch: initial conditions diverge

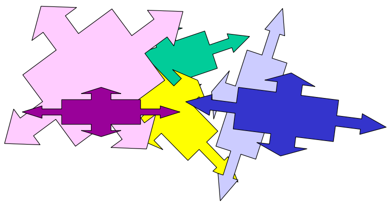
Fold: divergent conditions converge

Chaos vs. Chaos

- Vernacular meaning: a state of extreme confusion and disorder
- If we find chaos in the data we collect it likely implies low dimensionality… high degree of control and/or cooperation
  - managerial
  - adaptive
  - institutional
  - self-induced

Why is chaos interesting?

- rejection of hypothesis of randomness
- indicates number of state variables
- use it to monitor dynamics
- indication of need for nonlinear models
- healthy or unhealthy state of system
- may have short term prediction capability
- indicate when we can, cannot predict
- indicates sensitivity to initial conditions



High dimensional systems

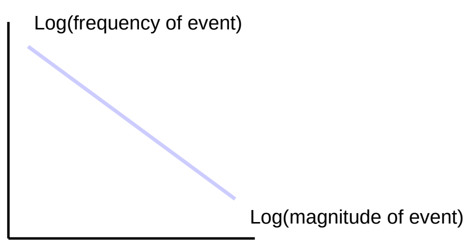
Inverse power law leads to colored noise

$$Y = A * X^{-B}$$
$$\text{Log } Y = \text{Log } A - B * \text{Log } (X)$$

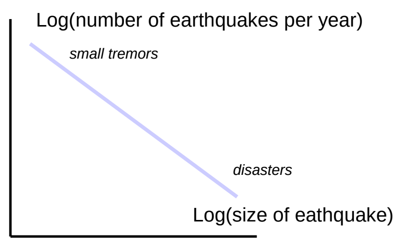
Where
- X and Y are two interrelated variables in the complex system
- A and B are constants

For example,
- X = magnitude of an event in a complex system
- Y = frequency of an event of particular size in a complex system

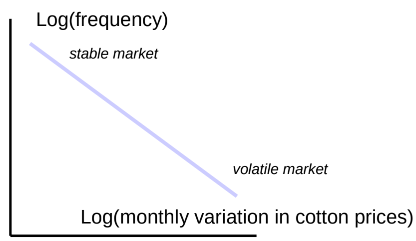
The Inverse Power Law: A Pervasive Model in Complex Systems



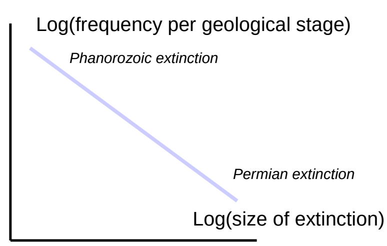
The Inverse Power Law: Example



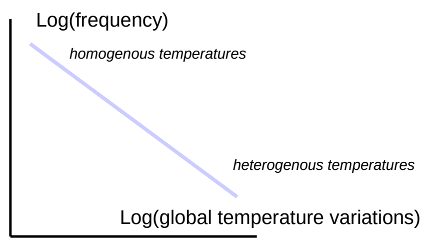
The Inverse Power Law: Example



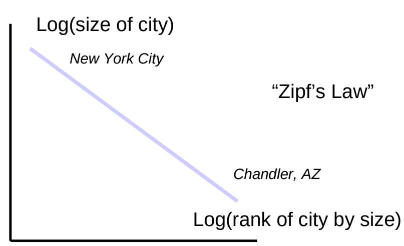
The Inverse Power Law: Example



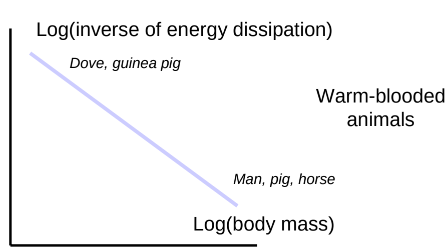
The Inverse Power Law: Example



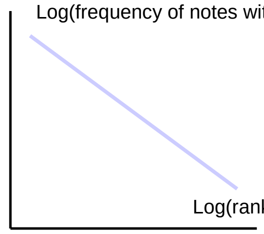
The Inverse Power Law: Example



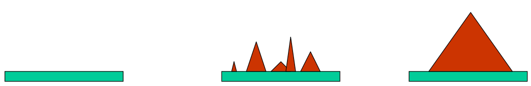
The Inverse Power Law: Example



The Inverse Power Law: Example



Bak's Sandpile

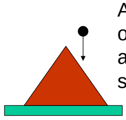


Empty table

Begin to drop grains of sand onto table top

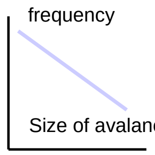
Sand self-organizes into sandpile

Sandpile avalanches



As I drop a single drop of sand onto the pile and measure the size of the avalanche...

Inverse Power Law/ Colored noise



Why? The sandpile has reached a point of "self-organized criticality", where every element is connected (ultimately) to every other element

Sandpile simulation

2	1	2	3
3	0	2	3
1	1	1	2
3	2	2	0

- Each cell on the grid has a height.
- As a drop of sand lands on the grid space, it increases the height.
- When it reaches a critical height, it collapses and sends sand to its neighbors.
- When this occurs we have an avalanche...

Initial configuration

Sandpile simulation

2	1	2	3
3	0	2	3
1	1	2	2
3	2	2	0

Grain of sand falls here; height increases from one to two

Avalanche size = 0

Sandpile simulation

2	1	2	4
3	0	2	3
1	1	2	2
3	2	2	0

Grain of sand falls here
4 is critical height, so...

Sandpile simulation

2	1	3	0
3	0	3	4
1	1	2	2
3	2	2	0

Sand collapses and adds to its neighbors' heights… and one of them reaches critical height…

Avalanche size = 1

Sandpile simulation

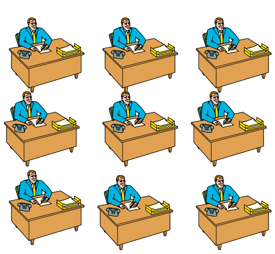
2	1	4	1
3	0	4	0
1	1	3	3
3	2	2	0

Sand collapses and adds to its neighbors' heights… and one of them reaches critical height…

Avalanche size = 2

ETCETERA...

The Sandpile organization



How big are the avalanches?

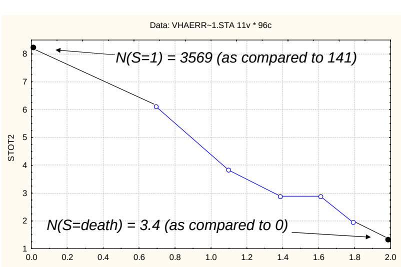
Amazing things...

- Universality of power law
- Emphasizes that simple, local interactions can create complex dynamics
- The generative mechanism behind small events IS THE SAME as that for large events

Medical error data

- Medical errors within five different hospitals
- March 1997 to March 1998
- Error type, date, and severity
  - 0 Potential error only
  - 1 Error occurred, no harm to patient
  - 2 Error occurred, increased monitoring only
  - 3 Error occurred, change in VS, additional labs, no permanent harm
  - 4 Error occurred, required additional treatment, increased LOS
  - 5 Error occurred, permanent harm to patient
  - 6 Error resulted in patient death
- "conventional" statistical perspective
- Linear patterns
- Severity and power laws

Power law for total



Conclusion

- Four types of dynamical systems
- Periodic
  - Few independent variables
  - Provides source of order
- Chaotic
  - Few interdependent variables
  - Divergence + convergence
- Colored noise
  - Many interdependent variables
  - Locally, cascading processes
- White noise
  - Many independent variables
  - Common versus special cause variation